

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS  
AS LEVEL  
H230/02  
MATHEMATICS A  
Pure Mathematics and Mechanics  
WEDNESDAY 23 MAY 2018:  
Morning  
TIME ALLOWED: 1 hour 30 minutes  
plus your additional time allowance  
MODIFIED ENLARGED 24pt**

**YOU MUST HAVE:**

**Printed Answer Booklet sent with the  
standard paper or any suitable paper  
provided by the centre. The Printed Answer  
Booklet may be enlarged by the centre.**

**YOU MAY USE:**

**a scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**



## **INSTRUCTIONS**

**Use black ink. HB pencil may be used for graphs and diagrams only.**

**Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number or write them on the paper provided.**

**Answer ALL the questions.**

**IF YOU USE THE PRINTED ANSWER BOOKLET WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.**

**Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**

**You are permitted to use a scientific or graphical calculator in this paper.**

**Final answers should be given to a degree of accuracy appropriate to the context.**

**The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .**

## **INFORMATION**

**The total number of marks for this paper is 75.**

**The marks for each question are shown in brackets [ ].**

**YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**

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## Formulae

### AS Level Mathematics A (H230)

#### Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_r a^{n-r} b^r + \dots + b^n$$

$$(n \in \mathbb{N}), \text{ where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

#### Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

#### The binomial distribution

$$\text{If } X \sim B(n, p) \text{ then } P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},$$

Mean of  $X$  is  $np$ , Variance of  $X$  is  $np(1-p)$

## Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

**SECTION A: Pure Mathematics**  
**Answer ALL the questions**

**1 In triangle  $ABC$ ,  $AB = 20$  cm and angle  $B = 45^\circ$ .**

**(i) Given that  $AC = 16$  cm, find the two possible values for angle  $C$ , correct to 1 decimal place. [4]**

**(ii) Given instead that the area of the triangle is  $75\sqrt{2}$  cm<sup>2</sup>, find  $BC$ . [2]**

**2 (i) The curve  $y = \frac{2}{3+x}$  is translated by four units in the positive  $x$ -direction. State the equation of the curve after it has been translated. [2]**

**(ii) Describe fully the single transformation that transforms the curve  $y = \frac{2}{3+x}$  to  $y = \frac{5}{3+x}$ . [2]**

**3 In each of the following cases choose one of the statements**

$$P \Rightarrow Q \quad P \Leftarrow Q \quad P \Leftrightarrow Q$$

**to describe the relationship between  $P$  and  $Q$ .**

**(i)  $P$ :  $y = 3x^5 - 4x^2 + 12x$**

**$Q$ :  $\frac{dy}{dx} = 15x^4 - 8x + 12$  [1]**

**(ii)  $P$ :  $x^5 - 32 = 0$  where  $x$  is real**

**$Q$ :  $x = 2$  [1]**

**(iii)  $P$ :  $\ln y < 0$**

**$Q$ :  $y < 1$  [1]**

- 4 (i) Express  $4x^2 - 12x + 11$  in the form  $a(x + b)^2 + c$ . [3]
- (ii) State the number of real roots of the equation  $4x^2 - 12x + 11 = 0$ . [1]
- (iii) Explain fully how the value of  $r$  is related to the number of real roots of the equation  $p(x + q)^2 + r = 0$  where  $p$ ,  $q$  and  $r$  are real constants and  $p > 0$ . [2]
- 5 In this question you must show detailed reasoning.

The line  $x + 5y = k$  is a tangent to the curve  $x^2 - 4y = 10$ . Find the value of the constant  $k$ . [5]

- 6 A pan of water is heated until it reaches  $100^{\circ}\text{C}$ . Once the water reaches  $100^{\circ}\text{C}$ , the heat is switched off and the temperature  $T^{\circ}\text{C}$  of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where  $t$  denotes the time, in minutes, after the heat is switched off and  $a$  and  $k$  are positive constants.

(i) Write down the value of  $a$ . [1]

(ii) Explain what the value of 25 represents in the equation  $T = 25 + ae^{-kt}$ . [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is  $15^{\circ}\text{C}$  per minute.

(iii) Calculate the value of  $k$ . [3]

(iv) Find the time taken for the temperature of the water to drop from  $100^{\circ}\text{C}$  to  $45^{\circ}\text{C}$ . [3]

(v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than  $100^{\circ}\text{C}$ . Suggest how the equation for the temperature as the water cools would be modified by this. [1]

7 (i) Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0. \quad [3]$$

(ii) Hence solve the equation

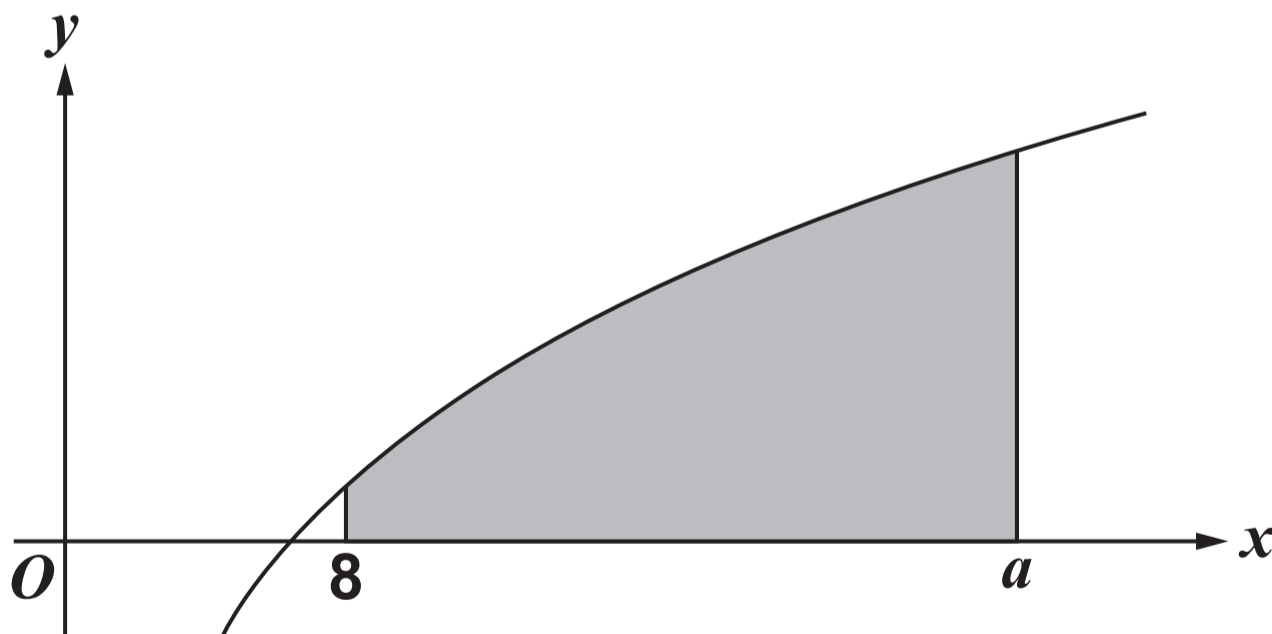
$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of  $\theta$  between  $0^\circ$  and  $180^\circ$ , correct to 1 decimal place. [5]

8 In this question you must show detailed reasoning.

The diagram below shows part of the graph of  $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$ .

The shaded region is enclosed by the curve, the  $x$ -axis and the lines  $x = 8$  and  $x = a$ , where  $a > 8$ .



Given that the area of the shaded region is 45 square units, find the value of  $a$ . [9]

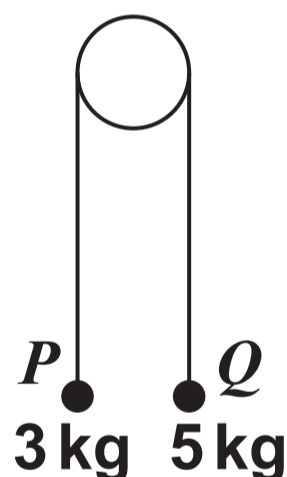
**SECTION B: Mechanics**  
**Answer ALL the questions**

- 9** In this question the horizontal unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in the directions east and north respectively.

A model ship of mass  $2\text{ kg}$  is moving so that its acceleration vector  $a\text{ m s}^{-2}$  at time  $t$  seconds is given by  $\mathbf{a} = 3(2t - 5)\mathbf{i} + 4\mathbf{j}$ . When  $t = T$ , the magnitude of the horizontal force acting on the ship is  $10\text{ N}$ .

Find the possible values of  $T$ . [4]

- 10** Particles  $P$  and  $Q$ , of masses  $3\text{ kg}$  and  $5\text{ kg}$  respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and  $P$  and  $Q$  are above a horizontal plane (see diagram below).

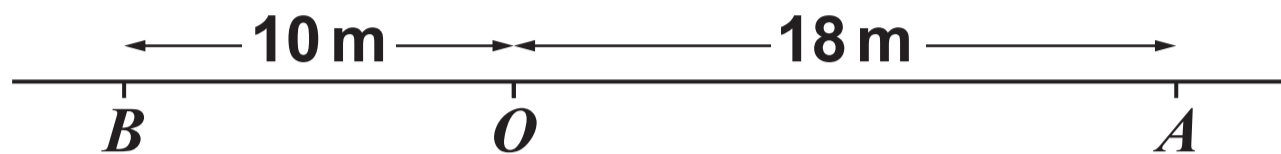


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- (i) Find the tension in the string immediately after the particles are released. [4]

After descending  $2.5\text{ m}$ ,  $Q$  strikes the plane and is immediately brought to rest. It is given that  $P$  does not reach the pulley in the subsequent motion.

- (ii) Find the distance travelled by  $P$  between the instant when  $Q$  strikes the plane and the instant when the string becomes taut again. [4]

- 11 A particle  $P$  is moving along a straight line with constant acceleration. Initially the particle is at  $O$ . After 9 s,  $P$  is at a point  $A$ , where  $OA = 18$  m (see diagram below) and the velocity of  $P$  at  $A$  is  $8 \text{ m s}^{-1}$  in the direction  $\overrightarrow{OA}$ .



- (i) (a) Show that the initial speed of  $P$  is  $4 \text{ m s}^{-1}$ . [2]

- (b) Find the acceleration of  $P$ . [2]

$B$  is a point on the line such that  $OB = 10$  m, as shown in the diagram.

- (ii) Show that  $P$  is never at point  $B$ . [4]

A second particle  $Q$  moves along the same straight line, but has variable acceleration. Initially  $Q$  is at  $O$ , and the displacement of  $Q$  from  $O$  at time  $t$  seconds is given by

$$x = at^3 + bt^2 + ct,$$

where  $a$ ,  $b$  and  $c$  are constants.

It is given that

the velocity and acceleration of  $Q$  at the point  $O$  are the same as those of  $P$  at  $O$ ,

$Q$  reaches the point  $A$  when  $t = 6$ .

- (iii) Find the velocity of  $Q$  at  $A$ . [5]

END OF QUESTION PAPER

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